THERMAL FEEDBACK IN TRANSISTOR OSCILLATORS

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***Abstract*—This paper deals with intrinsic thermal feedback (ITF) in transistor oscillators. ITF is an inherent property of any semiconductor device as it arises due to all basic material parameters are temperature dependent. The main topics under discussion are ITF impact on oscillator steady-state regime stability and noise.**

The usual approach to an oscillator analysis uses the only nonlinear differential equation describing first harmonic turns around a positive feedback loop (PFL). A role of an inertial unit plays the PFL resonator. A practical oscillator is controlled with at least two additional feedback loops: one describes its inertial auto-biasing network, another – inertial changes of transistor active zone temperature induced with dissipated power fluctuations. Thus, the oscillator steady-state regime (and its parameters) has to appear as a joint solution of the three nonlinear differential equations.

The corresponding analysis reveals conditions when ITF in conjunction with the inertial auto-biasing could upset the steady-state regime stability. The closer an oscillator to a threshold of unstability the more noise. In general case ITF acts like some effective built-in filter which colors additionally the noise spectra.

I. INTRODUCTION

Intrinsic thermal feedback (ITF) is an inherent property of any semiconductor device as it arises due to all basic material parameters are temperature dependent. By now, the ITF phenomenon is studied mostly in connection with self-heating effects in high-power transistors and their CAD models [1-4].

This paper deals with ITF in transistor oscillators, and particularly in those of them using bipolar junction transistors (BJT's). The topics under discussion are ITF impact on oscillator steady-state regime stability and noise.

The popular oscillator model developed by Leeson [5], which is based on consideration of the first harmonic turns around a closed oscillating loop, does not address all effects that are key to the prediction of oscillator performance. The authors missed to find out any publications devoted especially to ITF in oscillators with the exception of one old paper by D. Tsarapkin [6]. It demands to place more emphasis on a basic theory.

II. BASIC THEORY

A positive feedback loop (PFL) of a typical sine-wave oscillator comprises a sustaining amplifier (SA) and a band-pass filter (BPF) [5]. In the simplest case which is in consideration further SA is a single active device (BJT here) and BPF is some kind of a single tuned circuit connected as a transmission resonator.

The usual approach to an oscillator analysis considers first harmonic turns around PFL.

Put an alternating part of a BJT input voltage as

$$u(t) = U\cos(\omega t + \varphi_u) = \mathrm{Re}[\boldsymbol{U}\exp(j\omega t)] \qquad (1)$$

where U, ω and φ_u have a usual sense; $\boldsymbol{U} = U\exp(j\varphi_u)$ — a complex amplitude considering as a slow varying (in comparison with $\cos(\omega t)$) function of time.

Put similarly for an alternating part of a collector voltage

$$u_c(t) = U_c\cos(\omega t + \varphi_c) = \mathrm{Re}[\boldsymbol{U}_c\exp(j\omega t)]. \qquad (2)$$

$\boldsymbol{U}_c = U_c\exp(j\varphi_c)$ — a complex amplitude of the collector voltage is also a slow varying function of time. Its phase angle, φ_c, is a sum

$$\varphi_c = \varphi_{i1} + \varphi_z \qquad (3)$$

where φ_{i1} — a phase angle of the first harmonic, $\boldsymbol{I}_{c1}$, of an instantaneous collector current $i_c(t)$; φ_z — a phase angle of a collector load impedance, $\boldsymbol{Z}_c$.

In the shortened operator representation

$$\boldsymbol{Z}_c(\boldsymbol{p}) = R_c/(1 + \tau_c\boldsymbol{p}) \qquad (4)$$

where R_c — a resonant impedance of the tank (partly connected, in general); $\boldsymbol{p} = d/dt$— a differential operator along time; $\tau_r = Q/(\pi\nu_o)$ — a resonator time constant; ν_o, Q — a resonant frequency and a loaded Q-factor of the transmission resonator accordingly. In a steady-state regime

$$\boldsymbol{p} = j\Omega = j(\omega - \omega_o), \quad \omega_o = 2\pi\nu_o. \qquad (5)$$

The collector-to-base voltage transfer through BPF is described with a so-called *feedback coefficient*, $\boldsymbol{k}$. By definition,

$$\boldsymbol{k} = k\cdot\exp(j\varphi_k) = \boldsymbol{U}/\boldsymbol{U}_c. \qquad (6)$$

Put $\varphi_k = 0$ to simplify the problem. Then $\boldsymbol{k} = k$.

The next step is to choose a BJT model. This model should be as simple as possible to permit us to reveal the main features of the discussed phenomena. Arrange to ignore transistor parameters frequency dependence and a transistor input impedance influence on the oscillator

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circuitry. Then all needed for analysis is collector current volt-ampere characteristics (VAC's) and a thermal model.

As a matter of fact the instantaneous collector current, i_c, (as supposed, $i_c \cong i_e$) is a function of both the base-emitter, e_b, and collector-emitter, e_c, voltages: $i_c = i_c(e_b, e_c)$. For high enough e_c exceeding some critical value, $e_{c\text{-cr}}$, $i_c \approx i_c(e_b)$. Taking into account the voltage fall on a base resistance, r_b, one has

$$i_c = I_s\{\exp[q(e_b - i_c r_b)/(k_B T)] - 1\} \tag{7}$$

where I_s — an emitter saturation current; q — an absolute value of a charge on an electron; k_B — the Boltzmann's constant; T — a transistor die temperature in Kelvin.

A typical plot of (7) is drawn in Fig. 1a with a dashed curve. Bearing in mind an oscillator used to operate at a large signal, introduce a linear piece-wise approximation of VAC (solid lines in Fig. 1a) putting

$$i_c = i_{c\,(8)} = \begin{cases} 0, & \text{if } e_b < E', \\ S\cdot(e_b - E'), & \text{if } e_b \geq E'. \end{cases} \tag{8}$$

Here S — a forward transconductance in an active mode; E' — a *cut-off voltage*. This parameter is called also the *junction turn-on* or *knee voltage*. $E' = 0.6\ldots0.7$V in a case of Si BJT.

Put e_b as a sum of a biasing voltage, E_b, and a sinusoidal input signal $u(t)$ where temporarily $\varphi_u = 0$ for simplicity:

$$e_b = E_b + U\cos(\omega t). \tag{9}$$

Then $i_c(t)$ is a periodical function, and the collector current pulse itself typically looks like a piece of cosine (Fig. 1b). The parameter θ here referred to as a *conduction angle* describes a part of RF period corresponding i_c flow. Quantitatively,

$$\cos\theta = (E' - E_b)/U. \tag{10}$$

$\theta = \pi/2$ rad $= 90^o$ if $E_b = E'$ at an operating point. The harmonic content of the pulse in Fig. 1b is defined with

$$I_{c0(1)} = US\gamma_{0(1)}(\theta) \tag{11}$$

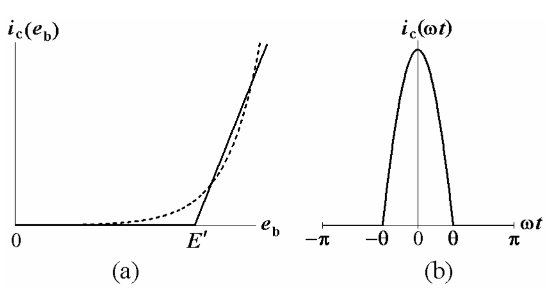

Figure 1. BJT VAC and its approximation (a) and a typical collector current form at a steady-state regime (b).

where $I_{c0(1)}$ — the appropriate current harmonic amplitudes; $\gamma_0(\theta) = (\sin\theta - \theta\cdot\cos\theta)/\pi$, $\gamma_1(\theta) = (2\theta - \sin2\theta)/(2\pi)$.

Fig. 2a demonstrates results of the transistor output VAC approximation. The horizontal lines correspond to (8). On the left they are confined with the *critical regime line* (CRL) obeyed the equation

$$i_{c\text{-cr}}(e_c) = S_{cr}\cdot e_c. \tag{12}$$

Define E_c as a collector-emitter bias. Then, obviously, $e_c(t) = E_c - U_c\cos(\omega t + \varphi_c)$, at that $\varphi_c = \varphi_u$, if $\varphi_{i1} = \varphi_u$ and $\boldsymbol{Z}_c = R_c$. Put these conditions met further.

During a working cycle

$$i_c(t) = \begin{cases} i_{c\,(8)}, & \text{if } i_{c\,(8)}[e_b(t)] \leq i_{c\text{-cr}}[e_c(t)], \\ i_{c\text{-cr}}, & \text{if } i_{c\text{-cr}}[e_c(t)] \leq i_{c\,(8)}[e_b(t)]. \end{cases} \tag{13}$$

The *critical regime* (CR) takes place when

$$i_{c\max} = i_{c\,(8)}(e_{b\max}) = i_{c\text{-cr}}(e_{c\min}), \tag{14}$$

i.e., i_c reaches maximum at the interfaces between CRL and one of the lines (8).

When U_c exceeds its critical value, $U_{c\text{-cr}}$, the current pulse acquires some depression at a centre (Fig. 2b). It is a characteristic sign of the *over-critical regime* (OCR). On the other hand, call a regime as the *pre-critical* one (PCR) if $U_c < U_{c\text{-cr}}$. As a rule, high quality oscillators must operate namely at PCR. All following calculations will suppose just the PCR case.

III. Main Equations

A steady-state regime of any oscillator has to meet four main closed loops of functional dependences:

– the first harmonic power balance;
– the biasing voltage balance;
– the thermal balance;
– the higher harmonics power balances.

In the following analysis we use the first three of them. Calculations are conducted in terms of an approach based on the symbolic shortened equations technique developed by

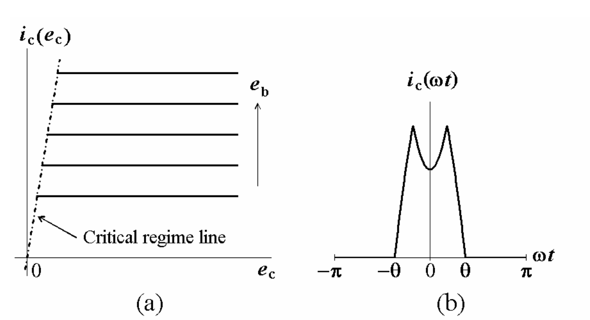

Figure 2. Approximation of the transistor output VAC (a) and a form of the collector current pulse in the over-critical regime (b).

S. I. Evtyanov [7]. The similar approach (quite restricted as compared to [7]) is popular under name of the Kurokawa's method.

The complex amplitude of an input voltage is

$$\boldsymbol{U} = \boldsymbol{Z}_{g}(\boldsymbol{p})\boldsymbol{I}_{c1}. \tag{15}$$

Here $\boldsymbol{Z}_g(\boldsymbol{p}) = \boldsymbol{k}\boldsymbol{Z}_c(\boldsymbol{p})$ — a *guiding impedance*. The reciprocal $\boldsymbol{Y}_g(\boldsymbol{p}) = 1/\boldsymbol{Z}_g(\boldsymbol{p})$ is called the guiding *admittance*. Thus, the first of the three equations is

$$\boldsymbol{U} - \boldsymbol{Z}_{g}(\boldsymbol{p})\boldsymbol{I}_{c1}(U, E_b, T) = 0. \tag{16}$$

Generally, $\boldsymbol{Z}_g(\boldsymbol{p})$ might be a ratio of two polynomials with complex coefficients. But here the applied simplest form (4) gives

$$\boldsymbol{Z}_{g}(\boldsymbol{p}) = R_g/(1 + \tau_r\boldsymbol{p}), \qquad R_g = kR_c. \tag{17}$$

which leads to a pair of equations for real variables:

$$\tau_r\dot{U} + U - R_g I_{c1}(U, E_b, T) = 0 \quad (\dot{U} \equiv \frac{dU}{dt}), \tag{18}$$

$$\frac{d\varphi_u}{dt} \equiv \dot{\varphi} = 0. \tag{19}$$

For a biasing circuit including an external source $E_{b\text{-}ex}$ and an emitter biasing resistor R_e bypassed with a capacitor C_e one has

$$E_b = E_{b\text{-}ex} - \boldsymbol{Z}_e(\boldsymbol{p})I_{c0}(U, E_b, T). \tag{20}$$

Here $\boldsymbol{Z}_e(\boldsymbol{p}) = 1/\boldsymbol{Y}_e(\boldsymbol{p}) = R_e/(1+\tau_b\boldsymbol{p})$ — the biasing emitter "resistance", $\tau_b = R_eC_e$ — a biasing circuit time constant.

Take the same simple *RC*-model for a thermal circuit putting a total thermal resistance of the package and the die, $\boldsymbol{Z}_T(\boldsymbol{p})$, like $\boldsymbol{Z}_T(\boldsymbol{p}) = 1/\boldsymbol{Y}_T(\boldsymbol{p}) = R_T/(1+\tau_T\boldsymbol{p})$, $\tau_T = R_TC_T$ — a thermal time constant. Then T is given by:

$$T = T_a + \boldsymbol{Z}_T(\boldsymbol{p})P_d \tag{21}$$

where T_a — an ambient temperature;
$P_d = P_0 - P_1$ — the power dissipation;
$P_0 = E_c \cdot I_{c0} = (E_{c0} - I_{c0}R_e)\cdot I_{c0}(U, E_b, T)$ — the power led to a transistor from a collector supplying source;
E_{c0} — a collector-emitter voltage from the supplying source, $E_c < E_{c0}$ owing to R_e, $I_{c0} > 0$;
$P_1 = ½U_c^2/R_c = ½U^2/(kR_g)$ — the first harmonic power at a collector output.

The equations set of (16), (20) was studied in [8, 9] and some other works by the Soviet period. In the next section we repeat some old results and analyse the new features arising due to adding the thermal balance (21).

IV. STEADY-STATE REGIME AND ITS STABILITY

In a steady-state regime (SSR) all amplitudes and an oscillation frequency are constant. It results the differential equations under analysis transforms to (19) describing an oscillation frequency shift in respect of a resonant frequency, $\Delta\omega = \omega\text{-}\omega_o$, and a set of the three algebraic transcendental equations:

$$U^o = \frac{E' - E_b^o}{\cos(\theta^o)}, \quad \gamma_1(\theta^o) = \frac{1}{G_0}, \quad G_0 = SR_g;$$

$$E_b^o = E_{b-ex} - R_e I_{c0}^o(U^o, E_b^o, T^o); \tag{22}$$

$$T^o = T_a + R_T[(E_{c0} - R_e I_{c0}^o) I_{c0}^o(U^o, E_b^o, T^o) - \frac{(U^o)^2}{2kR_g}].$$

Here the upper indices "o" identify SSR's values; $G_0 > 1$ — a regeneration factor, i. e., the open PFL voltage or current small-signal gain (in times).

The variables U and E_b are functionally bound in SSR. A plot of $U^o(E_b^o)$ on the UE_b-plate is called the *quenching diagram* (QD). In the PCR zone the QD family forms a fan of straight lines starting from the point $\{E', 0\}$ (Fig. 3). When some QD reaches CRL the curve turns to the right and becomes about a horizontal in the OCR zone. The CRL position is defined here with a formula

$$U_{cr} = \frac{E_c - (S/S_{cr})\cdot(E_b - E')}{S/S_{cr} + 1/k}. \tag{23}$$

Any point of a particular QD satisfies the first harmonic power balance (18). To define concretely the operating point one needs to insert some additional condition. This role takes a *biasing diagram* (BD), i. e., E_b versus U dependence in accordance with (20). The corresponding example for $G_0 \approx 3$ is shown in Fig. 3.

Note, for mild oscillation building-up the transistor has to operate in an active mode that turns into a cut-off one during a transition period. It means the initial biasing value, E_{b0} in Fig. 3, must exceed E'.

$G_0 = 2...5$ is the usual choice in practical design. However, the PCR parts of QDs can not be realized under external biasing in this case since they are principally

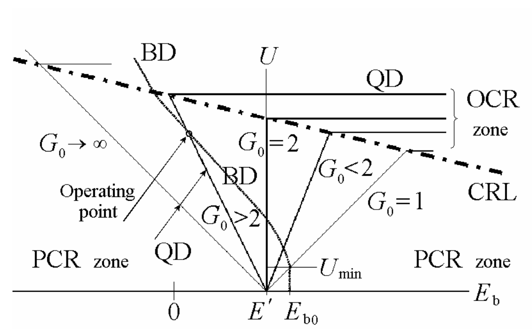

Figure 3. Graphical representation of main components of the SSR analysis including PCR/OCR zones, CRL, the quenching diagrams (solid lines) and one biasing diagram (a dotted line).

unstable as having a negative slope. Thus, using some kind of auto-biasing is often a forced solution.

Nevertheless, the operating points still stay potentially unstable when $G_0 \geq 2$ and they indeed become unstable if the biasing circuit inertia described quantitatively by τ_e exceeds some critical value [8].

Add small perturbations to our main variables to analyse SSR stability, i. e., replace

$$\varphi \rightarrow \varphi + \delta\varphi ,$$

$$U \rightarrow U^o + \delta U,\ E_b \rightarrow E_b{}^o + \delta E,\ T \rightarrow T^o + \delta T. \qquad (24)$$

Then it follows to a first approximation:

$$\boldsymbol{U} = (U^o + \delta U + j\cdot\delta\varphi\cdot U^o)\cdot\exp(j\varphi),$$

$$\boldsymbol{I}_{c1} = (I^o{}_{c1} + \sigma_{u1}\cdot\delta U + \sigma_{e1}\cdot\delta E + \sigma_{T1}\cdot\delta T + j\cdot\delta\varphi\cdot I^o{}_{c1})\cdot\exp(j\varphi),$$

$$I_{c0} = I^o{}_{c0} + \sigma_{u0}\cdot\delta U + \sigma_{e0}\cdot\delta E + \sigma_{T0}\cdot\delta T, \qquad (25)$$

where

$$\sigma_{u0(1)} = \frac{\partial I_{c0(1)}(U, E_b, T)}{\partial U}\Bigg|_{U^o, E_b^o, T^o},$$

$$\sigma_{e0(1)} = \frac{\partial I_{c0(1)}(U, E_b, T)}{\partial E_b}\Bigg|_{U^o, E_b^o, T^o},$$

$$\sigma_{T0(1)} = \frac{\partial I_{c0(1)}(U, E_b, T)}{\partial T}\Bigg|_{U^o, E_b^o, T^o} \qquad (26)$$

are pertinent partial derivatives taken at the SSR point.

$\dot{\varphi} \equiv 0$ in our simple oscillator mathematical model. Hence, the frequency correction $\Delta\dot{\varphi} \equiv 0$ as well and this item can be not considered further.

Substituting (25) to (18), (20), (21) and taking (22) into account, we arrive to a system of linear equations for disturbances:

$$\begin{pmatrix} \sigma_{u1} - \boldsymbol{Y}(\boldsymbol{p}) & \sigma_{e1} & \sigma_{T1} \\ \sigma_{u0} & \sigma_{e0} - \boldsymbol{Y}_e(\boldsymbol{p}) & \sigma_{T0} \\ \sigma_{u0}E_c - \dfrac{U}{kR_g} & \sigma_{e0}E_c & \sigma_{T0}E_c - \boldsymbol{Y}_T(\boldsymbol{p}) \end{pmatrix} \times \begin{pmatrix} \delta U \\ \delta E \\ \delta T \end{pmatrix} = 0 \qquad (27)$$

The steady state is stable if all roots of a characteristic equation det (27) = 0 have negative real parts.

The numerical calculations are made for BJT KT312A having at T_a = 300K, E_{cb} = 5V, I_{c0} = 7 mA:

f_t = 300 MHz, $I_{s0} = 4.9\cdot10^{-13}$ A, τ = 25 ps, $I_{c0\ max}$ = 30 mA,

E' = 0.6 V, S = 0.08 A/V, S/S_{cr} = 3, $P_{d\ max}$ = 225 mW, R_T = 0.6 K/mW.

Supposing an operating frequency is low enough (e. g., 1 MHz), choose $G_0 = SR_g = 3$. It follows, R_g = 37.5 Ω. Using the introduced approximations one gets θ = 1.303 rad.

If CR is to be realized then for $P_{1\text{-}cr}$ = 160mW, E_c = 12V:

U_c/E_c = 0.757, $U = U_{cr}$ = 1.32 V, I_{c1} = 35.2 mA, R_c = 258.1 Ω, k = 0.145; I_{c0} = 20.8 mA, E_b = 0.25 V.

Choose SR_e = 3 for the auto-biasing circuit as well. Then R_e = 37.5 Ω, and $E_{b\text{-}ex} = E_{b\text{-}cr} + R_e\cdot I_{c0\text{-}cr}$ = 1.03V.

Introduce the parameter

$$\zeta = U/U_{cr}, \qquad (28)$$

that is, normalize U in respect of its CR value, U_{cr}, to have a possibility to consider any point of the chosen QD.

These data are used as basic to calculate SSR at real T.

The self-heating effect is manifested itself mainly in reduction of cut-off voltage and the forward transconductance. To model such an effect, temperature correction is added as in the following:

$$\Delta E(T) = \Delta E_0 - K_T\cdot(T - T_0),$$

$$I_s(T) = I_{s0}\cdot\left(\frac{T}{T_0}\right)^3\cdot\exp\left[\frac{\Delta E(T)q}{k_B T_0}\cdot\left(1 - \frac{T_0}{T}\right)\right],$$

$$r_b(T) = r_{b0}\frac{T}{T_0} = \frac{I_{00} + I_{s0} - S_0 k_B T_0/q}{S_0(I_{00} + I_{s0})}\cdot\frac{T}{T_0},$$

$$S(T) = \frac{I_{00} + I_s(T)}{k_B T + r_b(T)(I_{00} + I_s(T))}\cdot q, \qquad (29)$$

$$E'(T) = U(I_{00}, T) - I_{00}/S(T).$$

Here: ΔE_0 = 1.11 V for Si, $K_T = 3\cdot10^{-4}$ V/deg, I_{00} = 18.6 mA — an approximation fitting parameter, r_{b0} = 11.12 Ω.

The natural time scale for our problem is a resonator time constant, τ_r. $\tau_r = 15.92\cdot10^{-6}$ s if ν = 1 MHz, Q = 50. Normalize τ_b and τ_T in respect of τ_r, thus introducing

$$\tau_{bn} = \tau_b/\tau_r,\ \ \tau_{Tn} = \tau_T/\tau_r. \qquad (30)$$

The SSR stable zone borders arising from the characteristic equation analysis are plotted in Fig. 4 (for $G_0 = SR_g = 3$) and in Fig. 5 (for G_0 = 1.8). The results demonstrate strong influence of the SSR parameters on the stable zone dimensions. In general, the more oscillation amplitude the greater a limit value of τ_{bn}, $\tau_{bn\ max}$. The thermal inertia can either increase or decrease the permitted value of τ_b depending on particular circumstances.

For many years it was considered SSR is stable unconditionally if $1 < G_o < 2$ [8, 9]. Our calculations revealed for the first time (Fig. 5) that, as a matter of fact, it is not

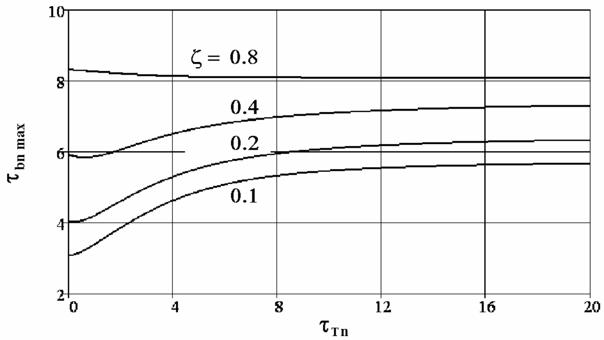

Figure 4. Influence of operating point position on the normalized biasing stability threshold vs. normalized thermal time constant.
$G_0 = SR_g = 3$; $SR_e = 3$; $P_{1cr} = 160$ mW; $E_{c0} = 12$ V.

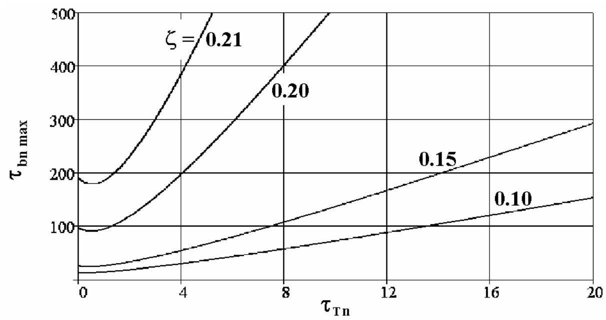

Figure 5. Influence of operating point position on the normalized biasing stability threshold vs. normalized thermal time constant.
$G_0 = SR_g = 1.8$; $SR_e = 3$; $P_{1cr} = 160$ mW; $E_{c0} = 12$ V.

true when thermal inertia is taken into account. There is the obvious threshold of stability with respect to τ_b unless ζ exceeds some critical value.

Choose $\zeta = 0.4$ as a particular SSR point. The phase-plane portrait in Fig. 6 and a temperature plot in Fig. 7 show the dynamics of oscillations building-up when SSR satisfies the stability conditions since $\tau_{bn} = 6$ here while $\tau_{bn\ max} = 7.22$ for $\tau_{Tn} = 15$. Note, the instant operating point stays into the OCR zone for an initial part of the transient.

Fig. 8 and Fig. 9, on the contrary, demonstrate situation outside the zone of stability. All three variables, U, E_b and

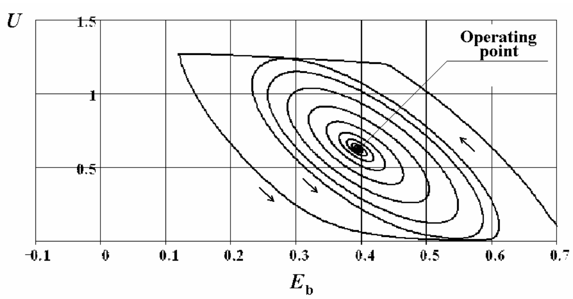

Figure 6. A phase-plane portrait of input amplitude vs. biasing voltage in case of a stable SSR point.
$G_0 = 3$; $SR_e = 3$; $P_{1cr} = 160$ mW; $E_{c0} = 12$ V; $\zeta = 0.4$; $\tau_{bn} = 6$; $\tau_{Tn} = 15$.

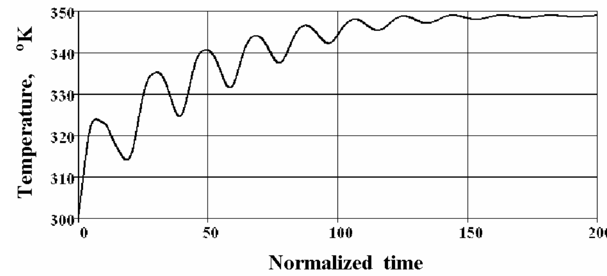

Figure 7. The temperature transient for the case in Fig. 6.

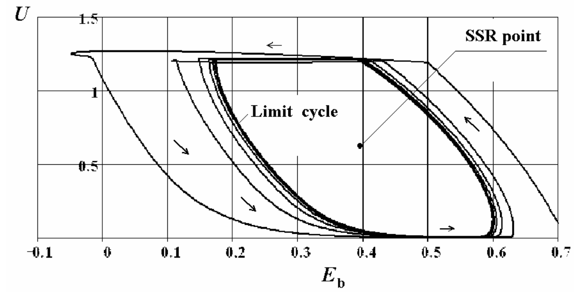

Figure 8. A phase-plane portrait of input amplitude vs. biasing voltage in case of an unstable SSR point.
$G_0 = 3$; $SR_e = 3$; $P_{1cr} = 160$ mW; $E_{c0} = 12$ V; $\zeta = 0.4$; $\tau_{bn} = 7.5$; $\tau_{Tn} = 15$.

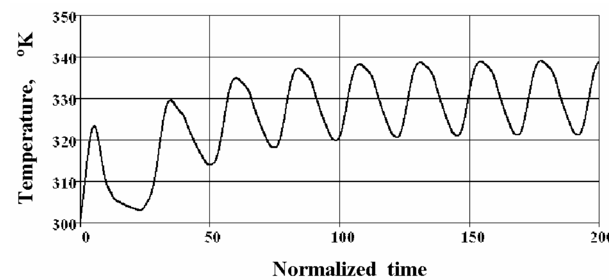

Figure 9. The temperature transient for the case in Fig. 8.

T, are subjected to self-modulation. In result they form a closed curve in 3D-space. Its projection on UE_b-plate displays a limit cycle around the SSR point. Note gradual decay of temperature variations during transient in Fig. 7 changing by periodic swings in Fig. 9.

V. Noise Analysis

Put collector current shot noise as the only noise source and omit non-linear noise transfer to simplify the problem. Then instead of (25) one has:

$$\boldsymbol{I}_{c1} = [(I^o_{c1} + \ldots + j\cdot\delta\varphi\cdot I^o_{c1}) + (I_{n1r} + j\, I_{n1i})]\cdot\exp(j\varphi),$$
$$I_{c0} = (I^o_{c0} + \sigma_{u0}\cdot\delta U + \sigma_{e0}\cdot\delta E + \sigma_{T0}\cdot\delta T) + I_{n0}, \qquad (31)$$

where I_{n1r}, I_{n1i} — real and imagine parts of the equivalent noise source around carrier, I_{n0} — the same for low-frequency noise.

According (19), (31), oscillator FM noise is defined with

$$\dot{\varphi}_n = \left(R_g / U^o \right) \cdot I_{n1i}. \quad (32)$$

It follows, PM noise is the same like in the Leeson's model.

To complete a system of noise equations it is sufficient to complete a right-hand side of (27) with a column vector

$$\begin{pmatrix} -I_{n1r} \\ -I_{n0} \\ -I_{n0} \cdot (E_{c0} - R_e I^o_{c0}) \end{pmatrix}. \quad (33)$$

Unfortunately, the formal solutions for fluctuations of main variables: U_n, $E_{(b)n}$, T_n, are too cumbersome to write them here. We replace them by Fig. 10,a-f where sets of frequency responses illustrate how approaching to threshold of stability modify the particular AM-noise. The possible noise rise can reach some tens decibel in the case of unhappy time constants choice. In Fig. 10, a-f $\alpha = (2Q/\nu) \cdot F$ — a normalized detuning of the transmission resonator at an offset frequency F; Q — a resonator loaded Q-factor.

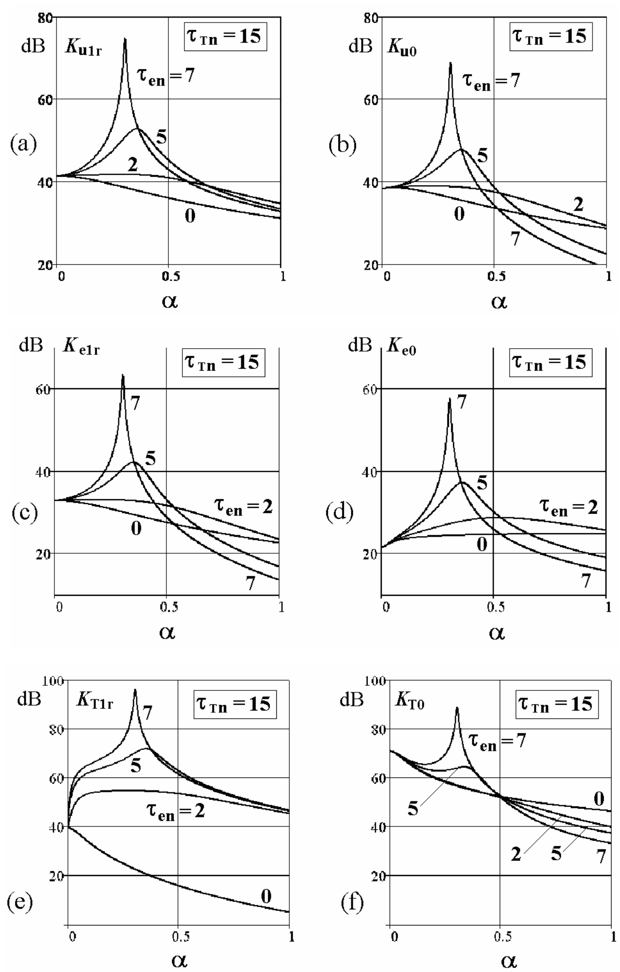

Figure 10. Impact of the biasing time constant on amplitude noise. $G_0 = 3$; $SR_e = 3$; $P_{1cr} = 160$ mW; $E_{c0} = 12$ V; $\zeta = 0.4$.

The transfer coefficients describe:

K_{u1r}, K_{u0}: SPD I_{n1r}, SPD $I_{n0} \rightarrow$ SPD of U;

K_{e1r}, K_{e0}: SPD I_{n1r}, SPD $I_{n0} \rightarrow$ SPD of E_b;

K_{T1r}, K_{T0}: SPD I_{n1r}, SPD $I_{n0} \rightarrow$ SPD of T.

Amplitude-to-phase, bias-to-phase, temperature-to-phase conversions are absent in the discussed simplest model. But they are definitely present of real transistors where

$$\varphi = \varphi(U, E_b, T). \quad (34)$$

It follows for SSR,

$$\varphi \rightarrow \varphi + \varphi_n = \varphi^o + \sigma_{\varphi u} \cdot U_n + \sigma_{\varphi e} \cdot E_n + \sigma_{\varphi T} \cdot T_n. \quad (35)$$

Here φ_n, U_n, E_n, T_n, $\sigma_{\varphi u}$, $\sigma_{\varphi e}$, $\sigma_{\varphi T}$ — noise perturbations and corresponding conversion coefficients accordingly.

With this remark one can hope the performed analysis gives useful insight into behavior of practical oscillators as any huge increase of amplitude fluctuations inevitably goes to up oscillator phase noise. More precise models will be considered in the future.

VI. Conclusion

In this paper we took into account the intrinsic thermal feedback in a bipolar transistor oscillator and found using simple models that this phenomenon together with inertia of auto-biasing circurtry affects sufficiently steady-state regime stability and oscillator noise. Lack of attention to this topic in practical design can result a lot of troubles.

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